Technical Comments

Comment on "Distributed Mass Matrix for Plate Element Bending"

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THE displacement functions for a rectangular plate element in bending, using the polynomial terms that obey the biharmonic equation, have now been published five times.[1–5] In view of the concluding remarks,[4] the plate considered evidently has uniform thickness, so that the mass matrix can be found explicitly. This has been done[5,7] and checked by the author, who added one instruction to a program intended for a quite different use. May one enquire whether the publication of such results is generally beneficial?

It is surprising to find explicit energy matrices in learned American journals when a large sector of the British airframe industry already has some such tedious work automated. The author proposed numerical integration independently some four years ago and now favors a technique that divorces from the main program the calculation of displacement functions and their derivatives at an integrating point, by writing it as a subroutine.

From this subroutine a rectangular matrix $[L]$ is assembled which operates on the nodal deflections to give the strains $\epsilon = Lx$, so that the strain energy is

$$\tfrac{1}{2}x'\textstyle\int L'DLx.d\,(\text{volume}) = \tfrac{1}{2}x'[\Sigma kJL'DL]x \qquad (1)$$

where k is the integrating constant resulting from multiple applications of a Gauss rule of integration, and J is a Jacobian; nontrivial if dimensionless coordinates are used in a quadrilateral, a triangle, or an element with a curved edge, for example. Kinetic energy, or second-order strain energy for stability calculations, is treated with little effort, as are different elastic problems that happen to use the same element geometry. Having found the nodal deflections, one can enter the subroutine again to find stresses, deflections between nodes, etc.

Thus, the programing effort, debugging costs, and checking are reduced, and the scope for experiment appears unlimited. In a simple case like that under discussion, the correct answer is found with few integrating terms, but as displacement functions become more complicated the choice of integrating accuracy must become an economic question.

Calculation of (1) is most cheaply accomplished[6] as follows:

$$\begin{bmatrix} -k^{-1}D^{-1} & L \\ L' & O \end{bmatrix} \begin{Bmatrix} \sigma \\ x \end{Bmatrix} \qquad (2)$$

where the matrix is symmetrically reduced as if notionally eliminating σ from a set of equations. Accumulation is automatic, or if extreme economy of storage is desirable one can accumulate directly into the assembled stiffness matrix. In any case, numerical integration techniques use very little storage. It is advisable to use the largest k last, to reduce roundoff error.

The author feels strongly that the research effort should be diverted to basic questions, such as the choice of displacement functions, the understanding of roundoff errors, etc.

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References

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Comment on "Magnetohydrodynamic Flow Past a Thin Airfoil"

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IN a previous paper[1] the Riemann invariants derived from the linearized system of Lunquist equations for general two-dimensional magnetohydrodynamic flow are given. Unfortunately, the solution for the invariants associated with the quadratic factor (in terms of the original notations of Ref. 1)

$$(\lambda \sin\theta + \cos\theta)^2 = M_0{}^2 \qquad (1)$$

which hold along transverse characteristics and exist only in general two-dimensional magnetohydrodynamic flow that $b \neq 0$ and $B_0 \neq 0$ [c.f. (11) of Ref. 1] is in error. The correct expression should read

$$\mathbf{r}_\pm = [1, 0, \pm(1/M_0), \mp(\lambda_\pm/M_0), (b/M_0{}^2), 0] + J_\pm \times [0, -\csc\theta, 0, 0, \pm(1/bM_0), 1/b] \qquad (2)$$

where

$$\lambda_\pm = (\pm M_0 - \cos\theta)\csc\theta \qquad (3)$$

as solved from (1) and

$$J_\pm = \cos\theta \pm M_0\{[\tan^2\theta(1 + \lambda_\pm{}^2)/(1 + \lambda_\pm \tan\theta)^2] - 1\} \qquad (4a)$$

which, by the use of (3), simplifies to

$$J_\pm = \pm(1/M_0) - \cos\theta \qquad (4b)$$

Substitution of $\lambda_\pm$ in (3) and $J_\pm$ in (4b) into (2) finally

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yields

$$\mathbf{r}_{\pm} = [1, 0, \pm(1/M_0], \pm[\cot\theta/M_0) - \csc\theta, b/M_0^2, 0] + [\pm (1/M_0) - \cos\theta)] \times [(0, - \csc\theta, 0, 0, \pm (1/bM_0), 1/b] \quad (5)$$

The corresponding Riemann invariants, $Z_{\pm} = \mathbf{r}_{\pm}\cdot\mathbf{P}$, hold along two families of straight transverse characteristics described by

$$dy/dx = \sin\theta\,(\cos\theta \mp M_0)^{-1} \quad (6a)$$

respectively, which, after integration, becomes

$$y = \sin\theta\,(\cos\theta \mp M_0)^{-1}x + C_{\pm} \quad (6b)$$

respectively, with $C_{\pm}$ being the integration constants or the parameters characterizing two families of straight characteristics, respectively.

It may also be pointed out that in Fig. 1 of Ref. 1 the correct expressions for t, $\tilde{A}_0^2$, and $\tilde{a}_0^2$ should read

$$t = (s^2 - 4a_0^2A_0^2)^{1/2} \qquad \tilde{A}_0^2 = (s - t)/2$$
$$\tilde{a}_0^2 = (s + t)/2 \quad (7)$$

respectively. An obvious misprint appears in (27) of Ref. 1 where the denominator of the integrand should read $\rho - x$.

Reference

[1] Cumberbatch, E., Sarason, L., and Weitzner, H., "Magnetohydrodynamic flow past a thin airfoil," AIAA J. 1, 679 (1963).

Comment on "Steadiness of a Plasmajet"

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In a recent article Pfender and Cremers[1] have noted that the inconsistency of temperature measurements obtained in various plasmajet acilities may be attributable to the unsteadiness of the arc column. They quote examples of the well-known blown arc and state that the spectrophotometric results may indicate excessively high temperatures if the arc column extends beyond the anode. The reason for this indication is that spectrographic equipment requires steady state conditions and is not capable of obtaining time resolved measurement. Thus, an unsteady oscillation of the order of 1 kc/sec would not be perceived and only an integrated recording of the spectral line intensities would be obtained.

The purpose of this comment is not to criticize the previous work but merely to point out that Harvey[2] et al., have communicated the existance of this behavior in a note that illustrated the temperature dependence on the source intensity. In that work, Harvey[2] has noted also that the arc instability occurs in nitrogen but not in argon gas, and that the hypothesis of a flexible current carrying conductor (viz., the arc column) is itself an unstable element. Subsequently, Simpkins[3] has shown that a similar instability of the arc column occurs for both helium and hydrogen operation, and that when such an instability occurs in the arc chamber, it manifests itself downstream of a settling chamber and a convergent-divergent nozzle. This result, which confirmed the earlier photodiode findings, was achieved by using an Acmade 35-mm drum camera to monitor the standoff distance of a bow shock wave created when a right circular cylinder was immersed in the test jet. These tests showed that the bow shock wave remained steady for the argon arc but exhibited an oscillatory motion when nitrogen operation was employed. A histogram using a sample of 100 consecutive frames from the drum camera is shown in Fig. 1, where it can be seen that the amplitude of the shock oscillation is some ±25% of the mean distance. Since the drum camera operation was limited to about 4500 frames/sec, only a qualitative comparison could be made with the arc instability frequency; however, within this limitation both sets of data were in agreement with the frequency level of 2 kc/sec.

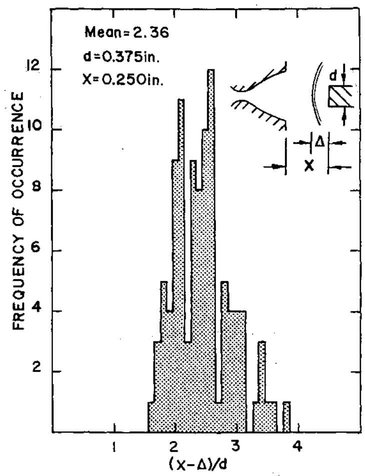

Fig. 1 Histogram of the shock standoff distance variation in an unstable nitrogen arc-jet.

The instability mentioned previously in the arc column is the result of a body force being exerted on the column when a local deformation takes place; the magnitude of which is of order I^2/R (I being the current and R the local radius of curvature). These phenomena have been examined experimentally by Curruthers and Davenport,[4] who have shown that, if the current density in the arc column becomes sufficiently large, an instability is spontaneously established. The possibility of such an effect occuring in a drawn arc facility has been discussed by Harvey et al., who have indications of such an onset when the mass flow rate of the gas has been decreased to a level where the vortex stabilization of the gas is insufficient to restrain the body force of the self-induced magnetic field.

It may be conjectured that the blown arc instability is re lated to its convective velocity, which is undoubtedly a function of the atomic weight of the gas being used. Therefore, one would expect that the arc column would become more stable as the atomic weight of the gas is increased. However, since there are large gradients and nonequilibrium conditions existing in the arc chamber, this hypothesis probably is oversimplifying the problem. Because of the instabilities described previously, caution must be exercised in interpreting any photographic data, since the time averaged measurements of the flow undoubtedly differ from the equivalent steady state measurements. It should be noted that time resolved temperature measurements can be obtained by coupling two or more photo-multipliers to a high dispersion spectograph.

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